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The Critical-Reflective Dimension of Ethnomodeling

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This discussion is related to critical-reflective dimensions of ethnomodeling, where educators create contexts that develop critical creativity in students and help them to practice applying a diversity of tools that enable them to solve problems they face in their own contexts. It is important to emphasize the critical-reflective dimensions of mathematical modeling in the context of active problem-solving situations. Ethnomodeling is a research paradigm related to critical-reflective dimensions of learning and which allows learners the opportunity to develop a sense of purpose and their own potential by using mathematics to examine and solve problems they deem important. In this paper, the author discusses the importance of both philosophical and theoretical perspectives found in critical-reflective dimensions in ethnomodeling as well as the importance for creating learning environments that assist students to develop their own critical-reflective abilities.

INTRODUCTION

Critical-reflective dimensions of ethnomodeling occur when educators create supportive environments that allow learners to develop their critical and to practice applying tools that enable them to solve increasingly complex problems faced in their own cultural context. This also allows them to develop competencies, abilities, and skills that focus on data and problems they are engaged in, indeed see as, worthy of exploration. In the past, this was done without engaging the community in the development, selection, planning and choosing of curricula. This aspect has contributed to a curricula and related educational system that creates cynical, unmotivated, disengaged and passive learners. The purpose of the paradigm, as outlined here, is to propose an alternative that engages both educators and students to become active, critical, and reflective participants in resolving conflicts in their own community or cultural context.

Conceptualizing Critical-Reflective Efficiency

One of the more important characteristics of a critical educator is the ability to develop processes that allow the analysis of data by learners that are related to day-to-day phenomena. Critical perspectives in relation to the social conditions that affect the students' own experiences, help to develop workable strategies needed to solve problems. This is a transformational form of learning, and is based on the context and previous experiences of learners. In this context, educators create conditions that help learners to challenge predominant opinions. By using their own context and experiences while learning to apply a data-based critical reflection to these problems, students are able to develop a rational discourse in order "to create meanings necessary for the structural transformation of society" (D'Ambrosio, 1990).

Rational discourse is therefore defined as the form of dialogue in which all parties have the same rights, responsibilities and duties to engage in, claim, and test the validity of their observations in an environment free of prejudice, fear, and social and political domination. It provides arguments for action and develops a workable plan that allows learners to transcend simple, often prejudical opinions not based on data. It gives learners the opportunity to enter into dialogue, resolve conflicts, and engage collaboratively. In this type of discourse, intellectual honesty, the elimination of prejudices, and the use of critical analysis based on data and facts, are important aspects that allow for dialogue to happen rationally and freely (Rosa and Orey, 2007). In this educational environment, the processes of discourse, conscious work, intuition, creativity, criticality, and

emotion are important elements that combine to help students develop their own critical-reflectiveness as they learn to focus on the data related to the problem.

The Theoretical Basis for a Critical-Reflective Dimension in Mathematical Modeling

Critically reflective teaching places both educators and learners at the center of the teaching and learning process. Classrooms are considered active laboratories where educators coach students to develop intuitive, creative and criticality by applying pedagogical approaches to real life situations. The act of teaching becomes a social and cultural activity that introduces students to the process of knowledge creation instead of passively receiving information (Freire, 2000).

Currently, the debate between conflicting teaching approaches continues mostly due to the over emphasis on ranking, evaluation and testing; where teaching is more or less centered on memorization and the testing of content. Simply put, I reject this approach. The need to elaborate a mathematics curriculum that promotes critical analysis, active participation, and reflection on social transformation by students (Rosa and Orey, 2007) is critical as humankind engages in resolving serious political, environmental and social problems that threaten all of us, everywhere. I am calling here for a curriculum change that prepares, indeed encourages, educators and learners to become critical, reflective, and responsible citizens, not as a by-product of standard curriculum and instructional practices, but as a fundamental goal for all learners. This aims to find practical solutions to real problems faced by society that actively engages learners to apply mathematics. It is impossible to teach mathematics or other curricular subjects in a way that is not neutral or insensitive to experiences of students (Fasheh, 1997).

Ethnomodeling is a teaching and learning methodology that focuses on critical-reflective efficiency that engages students in relevant and contextualized activities allowing them to be involved in the construction of their own mathematical knowledge. The theoretical basis for critical-reflective dimensions of ethnomodeling has its foundations in both Sociocultural Theory and the Critical Theory of Knowledge.

Sociocultural Theory

A fundamental aspect for this is through socialization, where knowledge is best constructed when students work in groups and learn to work cooperatively. Construction of knowledge is connected to other knowledge areas in a interdisciplinary manner. It is through real social interactions among students from diverse socio-cultural groups that learning is initiated and built (Vygotsky, 1986).

In ethnomodeling processes, diverse socio-cultural environments greatly influence student cognition. Collaborative work among educators and learners makes learning more effective because it generates higher levels of engagement in mathematical thinking through the use of socially and culturally relevant activities, and this makes use of *dialogical constructivism* because the source of knowledge is based on social interactions between students and environments in which cognition is the result of the use of *cultural artifacts* in these interactions. These artifacts act as vehicles allowing students to understand problems they face in their own community (Rosa and Orey 2007).

Critical Theory of Knowledge

Studies of Habermas' *Critical Theory of Knowledge* reinforce the importance of social contexts in the teaching and learning process. Habermas demonstrates how critical consciousness in learners is increased as they analyze social forces around them. This occurs through the use of strategies such as interpersonal communication, dialogue, discourse, critical questioning, and the use of problems taken from their own reality. The effects of social structure influence distinct knowledge areas taken by individuals from their social context. As well there are three generic knowledge domains: *technical*, *practical*, and *emancipatory* (Habermas, 1971).

Technical Knowledge or Prediction

The ability to predict, using technical knowledge, is defined by ways individuals manipulate their environmental contexts. It is gained through working with diverse empirical investigations and is governed by technical rules. In ethnomodeling processes, students learn to apply this as they collect data by coming to observe and document attributes of specific phenomena, verify if a specific outcome can be produced and reproduced, and know how to use rules to select different and efficient variables to manipulate and elaborate mathematical models (Brown, 1984). In this process, students improve their ability to communicate by using databased hermeneutics (written, verbal, and non-verbal communication) to verify social actions / norms modified by communication. It is here that meaning and interpretation of communicative patterns interact to construct and elaborate understandings that serve to outline agreements in social performance.

Emancipatory Knowledge or Criticism and Liberation

The process of gaining insight emancipates individuals from institutional forces that often limit and control their lives and is equally important to the development of any critical-reflective dimensions. How we come to determine our own unique opinions, answers and solutions to the social condition around us forms an essential objective. Knowledge is used to liberate individuals from outdated, often oppressive modes of social domination by developing the tools needed to exercise databased decision-making. In the ethnomodeling process, these insights are gained through mathematical modeling.

The act of ethnomodeling encourages learners to recognize what is needed to solve problems. Knowledge is gained by reflecting on the data that they themselves observe, collect, and analyze. Learning is linked to the growing technical knowledge of learners and the experiences gained in conjunction with social and cultural aspects through dialogical activities that enable an understanding of data collected. In the ethnomodeling process, this approach helps students to learn to take responsibility and/or ownership of their own knowledge and processes. Knowledge and ideas are then translated in interdisciplinary and dialogical ways as modelers begin to focus on the data used as instruments for social transformation.

Critical-Reflective Dimensions of Ethnomodeling

Currently, there is little consensus for specific epistemologies in the critical-reflective dimensions of ethnomodeling. It can describe it as a process that involves the elaboration, critical analysis, and validation of a model that represents a system taken from reality (Rosa and Orey, 2007). In this regard, ethnomodeling is an artistic process because in the process of elaboration of models, the modelers need to possess mathematical knowledge as well as intuition that allows for creativity as

they interpret their findings (Biembengut and Hein, 2000). These environments provide motivation as they develop and exercise mathematical creativity through critical analysis and the generation of knowledge. In this paradigm, students are coached and allowed to work from within their own reality and interests, and are allowed opportunities to actively learn and apply mathematical tools. The principal objectives of this approach:

1. Provide learners with mathematical tools necessary to study, act on, modify, change and transform their own reality.
2. Teach that deep learning of mathematics starts from the social and cultural contexts of students by providing them with opportunities to develop logical reasoning and creativity.
3. Facilitate the learning of mathematical concepts that help students build knowledge in mathematics so that they are able to understand their social, historical and cultural contexts.

Ethnomodeling forms a paradigm where learners are encouraged to describe, inquire about, and investigate problems coming from their immediate context, where they work with real problems that they have a personal connection to, and learn to use mathematics as a language for translating, understanding, simplifying, and solving problems. Modelers critically intervene in their own reality by obtaining a mathematical representation of the situation by means of reflective discussions related to the development and elaboration of the findings in their own models (Rosa and Orey, 2007). As they come to use mathematics to describe a setting, opinion, or problem-situation, modeling is akin to writing a mathematical poem. Critical-reflective dimensions of ethnomodeling are based on:

1. An increasing comprehension and understanding of reality in which learners live through reflection, critical analysis and actions based on data.
2. Learners borrow existing systems they study in the context of symbolic, systematic, analytical, and critical aspects.
3. Starting from given problem-situations, learners make hypotheses, test them, fix and improve upon them, draw inferences, generalize, analyze, conclude, and make decisions about the object under study.

From this perspective, ethnomodeling makes use of three gradually more complex mathematical modeling pedagogical phases (Barbosa, 2001).

Case 1: Educators Choose a Problem

In this type of practice, the teacher chooses the situation and then describes it for students, more often than not using preselected textbook examples. This is in accordance to curriculum content where the teacher provides students with the appropriate mathematical tools needed in the elaboration of classic mathematical models. In our experience, this is often the first step as learners learn to integrate ethnomodeling strategies in their experience (Rosa and Orey, 2007). These first experiences in many ways are related to Halpern's (1996) *critical thinking* that involves a range of thinking skills that lead toward desirable outcomes; and Dewey's (1933) *reflective thinking* that focuses on the process of making judgments about what has happened or was observed.

This approach allows students to solve classic problems, such as the study of the calculation of the circumference of the earth by Eratosthenes. By being shown how to describe a problem, organize variables and data, set up related equations to enable them to translate real situations into mathematical terms, they see how to make observation of patterns, testing of conjectures, and the eventual estimation of results. These are important processes for later stages as they become increasingly more and more autonomous and sophisticated in their ability to model. This most certainly introduces them to the process of mathematization, and allows students to construct and look at classic mathematical models.

Case 2: Educators Suggest and Elaborate the Initial Problem

At this stage, the teacher may give a common theme to the students, such as river pollution or transportation fares, and then modelers investigate the problem by collecting data, formulating independent hypotheses, and making necessary modifications in order to develop their mathematical models, that they share between groups. Students themselves are given more autonomy to participate in the activities proposed as they develop their own modeling awareness. One of the most important stages of this modeling process is related to the elaboration of a set of assumptions, which aims to simplify and solve the model to be developed (Rosa, Orey, and Reis, 2012).

Case 3: Educators Facilitate the Modeling Process and Engage in Ethnomodeling

Educators at this stage facilitate the modeling process by allowing students to choose and justify their own theme. The "ethnomodelers" are encouraged to develop a project in which they are responsible for all stages of the process: from justification for and formulation of the problem to its validation and final presentation. At this stage, supervision by the teacher is more like coaching, and involves encouragement in the modeling process. This enables a vital critical-reflective engagement in proposed activities by encouraging modelers to develop and justify their own hypotheses and opinions based on their data and research. Once again, this is very much like writing a poem or essay, at some point the learners must be freed from traditional limitations and the rote learning practices of mathematical grammar and prescribed textbook problems in order to practice developing their own prose and mathematical poems. During the development of ethnomodels, problems are chosen and suggested by the modelers themselves and are used to reflect critically on the aspects involved in the situation modeled. These are related to:

1. The diverse interdisciplinary connections they encounter;
2. Access and uses of many forms of technology; and
3. The discussion of environmental, economic, political, cultural, and social issues.

The use of mathematical content in this process is directed towards a critical databased analysis of problems faced by the members of their own community.

An Example of Ethnomodeling

The results from a conversation during a morning walk with students along a street in Ouro Preto encouraged exploration and development of a simple model that explored the relationship between mathematical ideas, procedures, and practices that developed connections between community members and formal academic mathematics. By observing the architecture of the façade of one of the schools in Ouro Preto, Brazil, this professor and his students were able to converse and explore

and eventually determine ways to relate functions of three types of curves: exponential, parabolic, and catenary to the patterns found on its wall (Rosa and Orey, 2013). After examining the data collected they measured various curves on the wall of the school and attempted to fit them to functions (exponential and quadratic) through mathematical models and came to the conclusion that the curves on the wall of the school closely approximated that of a catenary curve function.

Figure 1: Curves on the wall of the school (Photo by the author)

An Emancipatory Approach in the Critical-Reflective Dimension of Ethnomodeling

Because these pedagogical practices offer an open activity that allows us to apply multiple perspectives to solve a given problem, this is related to the emancipatory aspect of mathematics. However, the *open* nature of the (ethno) modeling activity may be difficult for students to establish and for them to develop models that satisfactorily represent the problem under study (Barbosa, 2001). Thus, the coaching, dialogical, and mediator role of educators is vital during the ethnomodeling process; this is why this approach is considered an extension of critical theories of knowledge, and forms the emancipatory aspects by addressing social-political issues.

According to the Brazilian National Curriculum for Mathematics (Brasil, 1998), all students must develop their own autonomous ability to gather data, solve problems, make decisions, work collaboratively, and communicate effectively. This cannot be done in environments that focus on uniform curriculum and testing. The approach as outlined here is based on developing *emancipatory powers*, where students are encouraged to become flexible, adaptive, reflective, critical, and creative citizens. This perspective is related to sociocultural dimensions of mathematics, and is directly associated with an ethnomathematics program (D'Ambrosio, 1990). It emphasizes the role of mathematics they see in their own context, by developing the ability to analyze a problem in relation to critical and reflective thinking aspects. As well, the role of modeling processes are used can be used to solve everyday challenges present in contemporary society.

Ethnomodeling may be understood as a language to study, understand, and comprehend problems faced daily by the community used to develop their own mathematical prose. For example, this approach is used to analyze, simplify, and solve daily phenomena in order to predict results or modify the characteristics of these phenomena (Bassanezzi, 2002). Figure 2 demonstrates this Critical-Reflective Ethnomodeling Cycle.

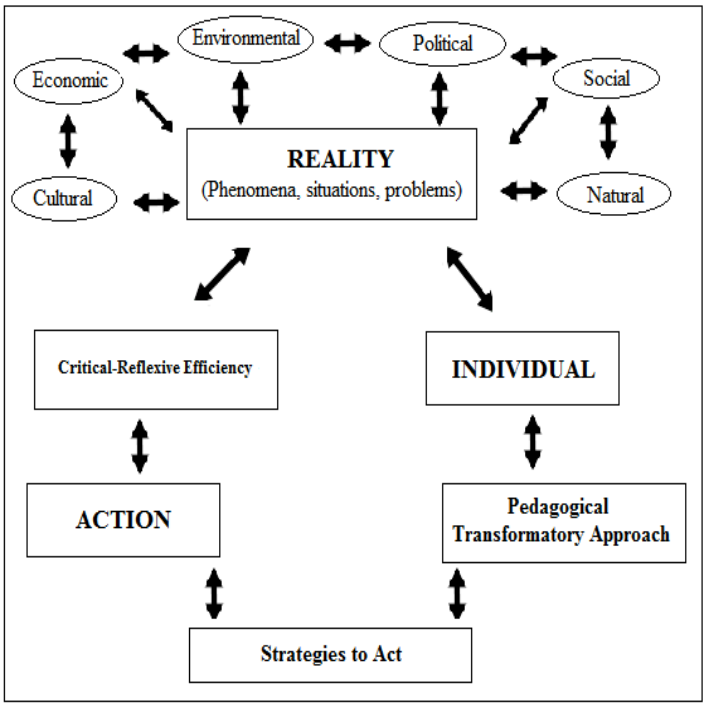

Figure 2: Critical-Reflective Ethnomodeling Cycle

Reflections on the reality modeled become a transforming action (Rosa and Orey, 2007), and this system allows modelers to make their own representations by using strategies that enable them to explain, understand, manage, analyze, and reflect their own unique problem-situation. The application of critically-reflective dimensions of modeling allows mathematics to be seen as a dynamic language used to explain phenomena and used to resolve important conflicts and problems; and is related to the reality of students as when they are passionately involved in sports or video gaming; where gamers deal with more and more abstractions, increasing in difficulty, and the development of and the creation of new tools, where the formulation of new concepts and theories allows them to solve (beat) the problem.

Final Considerations

Ethnomodeling is the process where by questioning of the themes or ideas developed are used to explain and make predictions related to the context (or phenomena) under study through the elaboration of models that represent these situations (Rosa and Orey, 2007). To ethnomodel then is related to the process that critically checks parameters selected for the solution of models gleaned from holistic interconnected contexts that the modelers themselves found valuable. It is not possible to explain, know, understand, manage, and cope with reality outside of realistic, interconnected and holistic contexts (D'Ambrosio, 1990). In the critically-reflective dimensions found in ethnomodeling, it is impossible to work only with isolated theories or techniques that facilitate models to be memorized, tested upon and quickly forgotten. Ethnomodeling gives access to creativity, conceptual elaboration, and the development of logical, reflective, and critical thinking and empowers modelers to connect what they learn to what they experience outside of school.

Fundamental characteristics for critical-reflective ethnomodeling are based on opinions stemming from databased analysis of problems students found valuable. The critical perspective of students in relation to social conditions that affect their own experience helps them to identify common problems and develop strategies used to solve them (D'Ambrosio, 1990). When the critical analysis of sociocultural phenomena, presents itself, ethnomodeling forms a teaching methodology that

focuses on the development of critical-reflective dimensions using contextualized teaching and learning (Rosa and Orey, 2007). Thus, ethnomodeling is based on the comprehension and understanding of reality of students, and makes use of reflection, analysis, and critical action. When students begin to study the symbolic, systematic, analytical and critical aspects, ethnomodeling can then explain different ways of working with reality. Critically-reflecting becomes a transformational action that reduces complexity of the reality by allowing students to explain it, understand it, manage it, and find solutions to the problems they themselves have developed.

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