Mathematical Modeling and its Critical-Reflexive Dimension

Abstract

Among the innovative teaching methodologies, it is important to highlight the use of critical-reflexive dimension of mathematical modeling approach to solve problem situations that afflict contemporary society. In the last three decades, mathematical modeling, particularly, research related to the critical-reflexive dimension of this approach has been looking for an identity, is defining its own objectives, and is developing a sense of its own nature and potential of research methods and investigations in order to legitimize pedagogical action. In this regard, it is necessary to discuss the importance of philosophical and theoretical perspectives found in the critical-reflexive dimension of mathematical modelling and its epistemology as well as the importance of a learning environment that helps students to develop their critical-reflexive efficiency.

Keywords: mathematical modeling, critical-reflexive dimension, sociocultural theory, social theory of knowledge, critical-reflexive efficiency, emancipatory approach.

Introduction

To begin we would like to reflect on critical-reflexive dimension of mathematical modeling. However, in order to discuss this issue, both inquiries are necessary:

- What is the role of schools in promoting of critical-reflexive efficiency in students?
- How do pedagogical practices currently used in the processes of teaching and learning mathematics impact critical-reflexive efficiency in students?

This context allows us to determine the main goals of schools that are related to the development of creativity and criticality in students that can help them to apply different tools to solve problems faced in their daily lives as well as to their competencies, abilities, and skills that help them reflect about problems faced by fellow members of their contemporary society, cultural group, or community.

Unfortunately, in most cases, these goals are established in the school curricula without the participation of communities in planning these actions. This curricular aspect may contribute to an authoritarian education whose main purpose is to promote demotivation and passivity in many students. Thus, the educational focus must be to prepare students to be active, critical, and reflective participants in society. However, in order to reach this goal, it is necessary that teachers promote teaching and learning processes that help students to develop critical-reflexive efficiency. This means that teachers should be adopting pedagogical practices that allow students to critically analyze problems that surround them in order for them to become active participants of society.

Conceptualizing Critical-Reflexive Efficiency

One of the most important characteristics of teaching for critical-reflexive efficiency is the emphasis on the critical analysis of students using the phenomena present in their daily lives. Another important feature of this kind of teaching is related to the students' reflections about social elements that underpin their globalized world. Thus, the critical perspectives in relation to social conditions that affect the students' own experiences help them to identify common problems and collectively develop strategies to solve these problems. This is a type of transformatory learning that is based on the previous experiences of students aims to create conditions that help them to challenge the worldviews and values predominant in society. In this regard, by using their own experiences and the critical reflection on these experiences, students are able to develop their own rational discourse in order to create meanings necessary for the structural transformation of society (D'Ambrosio, 1990).

Rational discourse is a special form of dialogue in which all parties have the same rights and duties to claim and test the validity of their arguments in an environment free of prejudice, fear, and social and political domination. In so doing, rational discourse provides an action plan that allows participants to enter into dialogue, resolve conflicts, and engage collaboratively to enable the resolution of problems in accordance to a set of specific rules. In this type of discourse, intellectual honesty, elimination of prejudices, and critical analysis of the facts are important aspects that allow dialogue to happen rationally (Rosa & Orey, 2007).

This context is related to the rational transformation that involves critical analysis of social phenomena. In this kind of educational environment, discourse, conscious work, intuition, creativity, criticality, and emotion are important elements that work to help students to develop their own critical-reflexive efficiency.

Teaching for Critical-Reflexive Efficiency

Education towards a critical-reflexive efficiency places students back at the center of the teaching and learning process. In this regard, classrooms are considered as learning environments in which teachers help, or coach, students to develop their creative and criticality by applying transformatory pedagogical approaches. However, in order for this form of pedagogy to be implemented in classrooms, it is necessary to discard transmissive traditional pedagogical approaches (Jennings, 1994). In other words, teaching is a social and cultural activity that should introduce students to the creation of knowledge instead of passively being recipients of its transmission. This means that the pedagogical transformatory approach is the antithesis of the pedagogical transmissive approach that seeks to transform students into containers filled with academic information in what Paulo Freire called a *banking mode education* (Freire, 2000).

Currently, the debate between these two teaching approaches continues, but the discussions are centered in relation to the contents to be taught and limited in relation to the time required to teach of these contents. Regarding this discussion, there is a need to elaborate a mathematics curriculum that promotes critical analysis, active participation, and reflection on social transformation by students (Rosa & Orey, 2007). There is a need for curriculum changes that seek to prepare students to become critical, reflexive, and responsible citizens. This mission aims to find practical solutions to the problem faced by society, which must be in accordance to the values and beliefs practiced by communities. This means that it is impossible to teach mathematics or other curricular subjects in a way that is neutral and insensitive to the reality experienced by students (Fasheh, 1997).

Thus, an important objective for schools in a democratic society is to provide necessary information through relevant activities so that students have the necessary tools to discuss and critically analyze curricular content by enabling them to solve daily problems and phenomena. In our point of view, mathematical modeling is a teaching methodology focused on critical-reflexive efficiency by students because it engages them in relevant and contextualized activities, which allow them to be involved in the construction of mathematical knowledge.

Theoretical Basis for the Critical-Reflexive Dimension of Mathematical Modeling

The theoretical basis for the social-critical dimension of mathematical modeling has its foundations in Sociocultural Theory and the Critical Theory of Knowledge.

Sociocultural Theory

Learning occurs through socialization because knowledge is better constructed when students work in groups by acting cooperatively in order to support and encourage each other. This approach allows students to reflect on complex problems embedded in authentic situations that help them to construct their knowledge by connecting it to other knowledge areas in an interdisciplinary way. According to this perspective, individuals' engagement in a sociocultural environment helps them to be involved in meaningful and complex activities. It is through social interaction (Vygotsky, 1986) among students from distinct cultural groups that learning is initiated and established. However, it is important to highlight that learning is triggered according to the purpose of each student because they have different capacity to act, react, reflect and, change their own environment in order to strategically transform it.

Thus, in the mathematical modeling process, social environments also influence student cognition in ways that are related to their own cultural context. In this learning environment, collaborative work among teachers and students makes learning more effective because generates high levels of mathematical thinking through the use of activities socially and culturally relevant This context allows the use of *dialogical constructivism* because the source of knowledge is based on social interactions between students and environments in which cognition is the result of the use of cultural artifacts in these interactions. Thus, these artifacts act as vehicles to help students to internalize changes by allowing them to understand social difficulties faced by the members of their own community (Rosa & Orey 2007).

Critical Theory of Knowledge

Studies of Habermas' *Critical Theory of Knowledge* reinforces the importance of social context for the teaching and learning process because this theory promotes the development of students' critical consciousness so that they are able to analyze how social forces shape their lives. This analysis occurs through intellectual strategies such as interpersonal communication, dialogue, discourse, critical questionings, and proposition of problems taken from reality.

The effects of social structure influence distinct knowledge areas that are purchased by individuals in the social environment. These areas are partly determined by interests that stimulate and motivate these individuals. Thus, in this theory it is recognized that there are three generic knowledge domains named *technical*, *practical* and *emancipatory* (Habermas, 1971).

Technical Knowledge or Prediction

It is defined by the way individuals control and manipulate the environment. It is gained through empirical investigations and governed by technical rules. In the mathematical modeling process, students apply this instrumental action when they observe the attributes of specific phenomena, verify if a specific outcome can be produced and reproduced, and know how to use rules to select different and efficient variables to manipulate and elaborate mathematical models (Brown, 1984).

Practical Knowledge or interpretation and understanding

It identifies individuals' social interaction through communication. In the mathematical modeling process, students communicate by using hermeneutics (written, verbal, and non-verbal communication) to verify if social actions and norms are modified by communication. It is in this kind of knowledge that meaning and interpretation of communicative patterns interact to construct and elaborate the community understanding that serves to outline the legal agreement for the social performance.

Emancipatory Knowledge or criticism and liberation

It is defined by the acquisition of insights that seek to emancipate individuals from institutional forces that limit and control their lives. It is necessary to determine social conditions that cause misunderstandings in the communication process, tactics that may be used to release particular oppressive and repressive forces, and risks that are involved in these tactics. The objective of this kind of knowledge is to emancipate individuals from diverse modes of social domination. In the mathematical modeling process, insights gained through critical self-awareness of the elaboration of mathematical models are emancipatory in the sense that students may be able to recognize the correct reasons to solve problems faced by their communities. During this process, knowledge is gained by self-emancipation through reflection leading to a transformed consciousness.

However, learning begins to be generated in the technical knowledge in conjunction with the social existence through interactive and dialogical activities. In the mathematical modeling process, this approach helps students to take ownership of the emancipatory knowledge. In this perspective, knowledge is translated in an interdisciplinary and dialogical ways so they can be used as instruments for social transformation.

Determining an Epistemology of the Critical-Reflexive Dimension of Mathematical Modeling

Currently, there is no general consensus on a specific epistemology for the critical-reflexive dimension of mathematical modeling. However, it can be described as a process that involves the elaboration, critical analysis, and validation of a model that represents a system taken from reality. In this regard, mathematical modeling could be considered as an artistic process because in the process of elaboration of a model, the modeler needs to possess mathematical knowledge as well as a dose of significant intuition and creativity to be able to interpret its context (Biembengut & Hein, 2000). In so doing, students need to work in a learning environment that provides necessary motivation so that they develop and exercise their creativity through critical analysis and the generation and production of knowledge. Research on the critical-reflexive dimension of mathematical modelling has defined its goals by establishing the nature and potential of their research methods and investigation. In this dimension, the junction of theory and practice assists students in understanding systems taken from their own reality to acquire the tools they need to exercise their citizenship and to actively participate in society.

The main objectives of this approach are:

- Provide students with the mathematical-pedagogical tools necessary to act, modify, change and transform their own reality.
- Teach that learning mathematics starts from the social and cultural context of the students by providing them with the opportunity to develop their logical reasoning and creativity.
- Facilitate the learning of mathematical concepts that help students build their knowledge in mathematics so that they are able to understand the social, historical and cultural context in which they live.

The use of critical-reflexive modelling dimension is based on:

- Comprehension and understanding of reality in which students live through reflection, critical analysis and critical action.
- When students borrow existing systems they study them in symbolic, systematic, analytical and critical ways.
- Starting from a given problem-situation, students are able to make hypotheses, test them, fix them, draw inferences, generalize, analyze, conclude, and make decisions about the object under study.

According to this context, mathematical modeling is considered as the paradigm for a learning environment in which students are invited indeed encouraged through the use of mathematics, to inquire and investigate problems that come from other diverse areas of reality. In this learning environment, students work with real problems by using mathematics as a language for understanding, simplifying, and solving these situations in an interdisciplinary fashion (Bassanezi, 2002). This means that mathematical modeling is a method of applied mathematics that was seized and transposed the field of teaching and learning as one of the ways to use reality in the mathematics curriculum. This enables them to intervene in their reality by obtaining a mathematical representation of the given situation by means of reflective and critical discussions on the development and elaborations of mathematical models (Rosa & Orey, 2007).

From this educational paradigm, there are three distinct mathematical modeling pedagogical practices that may be used in school curricula (Barbosa, 2001).

Case 1: Teachers Choose a Problem

In this pedagogical practice, teachers choose a situation or a phenomenon and then describe it to the students. According to the curriculum content to be developed, teachers provide students with necessary mathematical tools that are suitable to the elaboration of the mathematical models in order to solve the proposed problem. In our opinion, this is the first step to integrate mathematical modeling into the teaching and learning processes. However, for the development of the students' social-critical efficiency, there is also a need for active involvement in the process of teaching and learning mathematics (Rosa & Orey, 2007).

For example, in order to determine the height of a cliff, teachers choose a problem, situation or a phenomenon and then describe it to the students. This example is related to the *pragmatic* perspective of modelling. Mathematics is used in order to stimulate students' skills by using problem solving techniques during the modelling process. This perspective is also named *realistic* because problems and situations are authentic since they are also taken from other knowledge areas. It aims to enable the development of students' skills to solve problems that emerge during the mathematical modelling process (Shiraman & Kaiser, 2006).

By considering a typical exercise given in trigonometry: *From the top of a cliff, whose height is 100 m, a person sees a ship under a depression angle of 30°. Approximately, how far is the ship from the cliff?* (Rosa, Orey, & Reis, 2012) students can use the tangent function, *tg30° = 100/d*, in order to determine the distance from the base of the cliff to the ship.

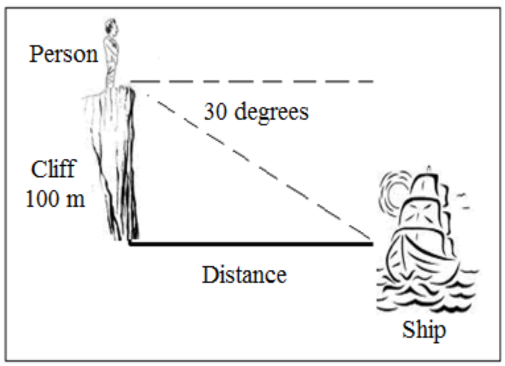

Figure 1: Representation of the problem presented by the teacher.

For many teachers, this trigonometric equation represents a simple mathematical model that demonstrates an application of trigonometry that illustrates the use of mathematics to solve a problem situation that may occur in reality. It is important to discuss with the students the assumptions that have been previously established as a critical analysis of the solution because this is an important aspect of the construction of mathematical models. During the process of the mathematization of this problem, some generalized simplifications of reality were established that are not critically discussed nor reflected with the students.

In this process of problem solving, it is assumed that the ocean is flat, the cliff is perfectly vertical to the straight line chosen to represent the distance from the base of the cliff to the ship, a straight line can reasonably approximate the distance from the base of the cliff to the ship, and he curvature of the Earth is ignored. On a small scale, this fact is not problematic, however, in a large-scale it can lead to significant deviations in the process of preparation and resolution of the mathematical model. It is also assumed that the height of the person is approximately equal to 1.70 meters, which is negligible compared to the height of the cliff, which is 100 meters; the angle of depression was exactly measured, and the ship is a significant distance from the cliff.

In this regard, a point can reasonably represent the position of the ship in the ocean. However, this point can get another mathematical meaning if the ship gets closer to the cliff. These assumptions are considered logical to simplify the problem because it provides a reasonable estimate to determine the distance between the base of the cliff and the ship. It is important to discuss with students that the answers to this type of problems or situations are never absolutely accurate. The analysis of mathematical models allows students to determine an accurate solution by using a detailed representation of reality.

These assumptions are related to Halpern's (1996) *critical thinking* that involves a wide range of thinking skills leading toward desirable outcomes and Dewey's (1933) *reflective thinking* that focuses on the process of making judgments about what has happened. This approach allows students to solve word problems by setting up equations in which they translate a real situation into mathematical terms, involve observation of patterns, testing of conjectures, and estimation of results, and helps students to mathematize systems taken from their own reality. Allow students to construct distinct mathematical models.

Case 2: Teachers Suggest and Elaborate the Initial Problem

In this pedagogical practice, students need to investigate the problem by collecting data, formulate hypotheses, and make necessary modifications in order to develop the mathematical model. Students are responsible for conducting the activities proposed in order to develop the modeling process. One of the most important stages of the modeling process refers to the elaboration of the set of assumptions, which aims to simplify and solve the mathematical model to be developed. In order to work with activities based on the social-critical dimension of mathematical modeling, it is necessary that students relate these activities to problems faced by their community (Rosa, Orey, & Reis, 2012).

For example, it is possible that teachers propose problems or questions that students can investigate similar to the following situation: *A company discharges its effluent into a river located near their facilities. These waters contain dissolved chemical substances that can affect the environment in which the river flows. How can we determine the concentration of pollutants in that river? How can we make sure that pollutant concentrations in the river is below the standard limit allowed by law?*

Students then investigated the problem by collecting data and were responsible for conducting activities proposed in order to develop the modeling process. One of the most important stages of the modeling process referred to the elaboration of the set of assumptions, which aimed to simplify and solve mathematical models to be elaborated as well as the development of a critical reflection on the data that will be collected.

In this context, it is important to discuss with the students certain modeling variables or constants, for example that:

1. The average velocity and rate of water flow was constant;
2. There is no seasonal change in the water level of the river
3. The rate of pollutant concentration in the river was constant;
4. The pollutant and the water are completely miscible regardless of the seasonal change in temperature; t
5. That there was no further precipitation during the period of data collection;
6. The pollutant and water mix completely;
7. The pollutant does not solidify in the sediments of the river;
8. The solid particles were deposited in the sediments of the river;
9. The pollutant is volatile because it can be reduced to gas or vapor at ambient temperatures;
10. The pollutant is chemically reactive

11. The shape of the river bed was uneven.

It is necessary to determine the key questions that affected the final concentration of pollutants in the river, as well as the rate of flow of pollutants on its waters. This activity helped students to reflect on the mathematical aspects involved in this problem, enabling them to understand phenomenon they encounter in their daily lives so they can critically solve a situation by focusing on the data and the using mathematics to resolve conflict.

Case 3: Teachers facilitates the mathematical modeling process

In this pedagogical practice teachers facilitate the mathematical modeling process by allowing students to choose a theme that is interesting to the members of the study group. Then, students are encouraged to develop a project in which they are responsible for all stages of the process, that is, from formulation of the problem to the validation of the solution. The supervision of the teachers is constant during the mediation of the teaching and learning process. This process enables students' social-critical engagement in the proposed activities.

However, even though there may be some disagreement regarding the use of a specific mathematical modeling pedagogical practice, it is possible to conduct activities, experiments, investigations, simulations, and research projects that interest and stimulate students at all educational levels. Thus, the choice of a pedagogical practice is to be used by teachers depends on the content involved, the maturity level of the students and the teachers' experience with the use of mathematical modeling process in the classroom. On the other hand, we emphasize that the critical analysis of the results obtained in either approach must be highly encouraged and developed.

During the development of the mathematical modeling processes, problems chosen and suggested by teachers or selected by students must be used to get them to critically reflect on all aspects involved in the situation to be modeled. These aspects are related to interdisciplinary connections, use of technology, and the discussion of environmental, economic, political, and social issues. Thus, the use of mathematical content in this process is directed towards the critical analysis of the problems faced by the members of the community.

For example, the results from a conversation during a morning walk with students along a street in Ouro Preto, Brazil encouraged exploration and developed some simple models and explored the relationships between mathematical ideas, procedures, and practices by developing connections between community members and formal academic mathematics. By observing the architecture of the façade of the school, professors and students were able to converse and explore and to determine ways to relate functions of three types of curves: exponential, parabolic, and catenary to the patterns found on its wall (Rosa & Orey, 2013).

Further, by analyzing these shapes, they observed how the curves on the wall were similar to exponential, parabolic, or catenary curves. However, when they visualized the shapes in only one part of the curve on the wall, they observed the existence of similarities between the exponential curves, parabolas, and catenaries. After examining the data collected when they measured various curves on the wall of the school and attempted to fit them to the exponential and quadratic functions through mathematical models they came to the conclusion that the curves on the wall of the school closely approximated a catenary curve function.

Figure 3: Curves on the wall of the school

The reflective aspect of this dimension is related to the emancipatory approach of the mathematics curriculum because its pedagogical practices offer *open* curricular activities that apply multiple perspectives to solve given problems, which require constant critical reflection on these solutions. However, the *open* nature of modeling activities may be difficult for students to establish and develop a model that satisfactorily represents the problem under study (Barbosa, 2001). Thus, the dialogical and mediator role of the teachers is very important during the modeling process.

The Emancipatory Approach of the Critical-Reflexive Dimension of Mathematical Modeling

The critical-reflexive dimension of mathematical modeling may be considered as an extension of the Critical Theory of Knowledge. In this regard, the emancipatory approach directs the educational objectives by addressing social and political issues in the pedagogical practices used in educational systems.

According to the Brazilian National Curriculum for Mathematics (Brazil, 1998), students need to develop their ability to solve problems, make decisions, work collaboratively, and communicate effectively. This approach is based on emancipatory powers, which helps students face challenges posed by society by turning them into flexible, adaptive, reflexive, critical, and creative citizens. This perspective is also related to the sociocultural dimensions of mathematics, which are closely associated with an ethnomathematics program (D'Ambrosio, 1990). This aspect emphasizes the role of mathematics in society by highlighting the necessity to analyze the role of critical and reflexive thinking about the nature of mathematical models as well as the role of the modeling process to solve everyday challenges present in the contemporary society.

The Process of the Critical-Reflexive Dimension of Mathematical Modeling

Mathematical modeling provides concrete opportunities for students to discuss about the role of mathematics as well as the nature of mathematical models so they can study systems taken from reality (Shiraman & Kaiser, 2006). It may be understood as a language to study, understand, and comprehend problems faced daily by the community. For example, mathematical modeling is used to analyze, simplify, and solve daily phenomena in order to predict results or modify the characteristics of these phenomena (Bassanezzi, 2002). Figure 2 shows the Critical-Reflexive Mathematical Modeling Cycle.

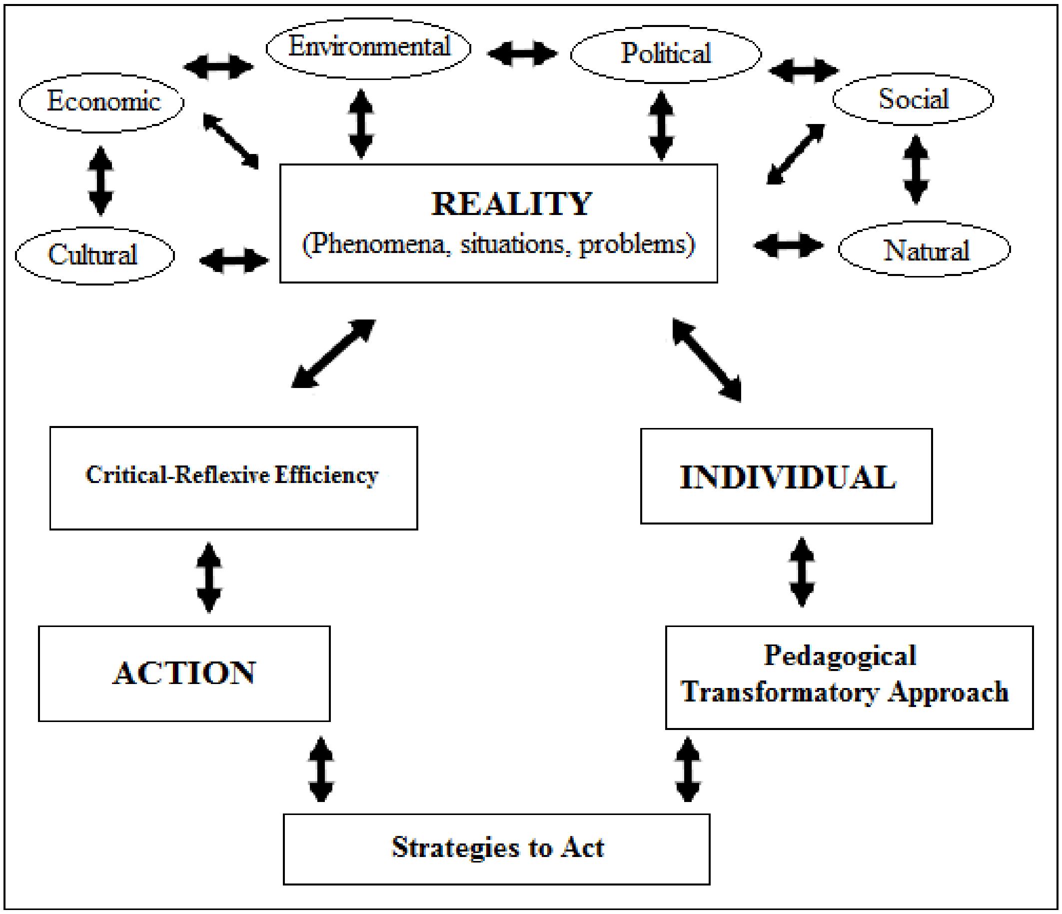

Figure 2: Critical-Reflexive mathematical modeling cycle.

In this process, the purpose of mathematical modeling is to develop students' critical and reflexive skills that enable them to analyze and interpret data, to formulate and test hypotheses, and to develop and verify the effectiveness of the mathematical models. In so doing, the reflection on the reality becomes a transforming action, which seeks to reduce the degree of complexity of reality through the choice of a system that it represents (Rosa & Orey, 2007). This isolated system allows students to make representations of this reality by developing strategies that enable them to explain, understand, manage, analyze, and reflect on all parts of this system. This process aims to optimize pedagogical conditions for teaching so that students are better able to understand a particular phenomenon in order to act effectively on this phenomenon to transform it according to the needs the community.

The application of critical-reflexive dimensions of mathematical modeling makes mathematics to be seen as a dynamic and humanized subject. This process fosters abstraction, the creation of new mathematical tools, and the formulation of new concepts and theories. Thus, an effective way to introduce students to mathematical modeling in order to lead them towards the understanding of its social-critical dimension is to expose them to a wide variety of problems or themes. As part of this process, questionings about the themes are used to explain or make predictions about the phenomena under study through the elaboration of mathematical models that represent these situations (Rosa & Orey, 2007).

However, the elaboration of mathematical models does not mean to develop a set of variables that are qualitative representations or quantitative analysis of the system because models are understood as approximations of reality. In this direction, to model is a process that checks whether the parameters are critically selected for the solution of the models in accordance to the interrelationship of selected variables from holistic contexts of reality. It is not possible to explain, know, understand, manage, and cope with reality outside the holistic context (D'Ambrosio, 1990). In the critical-reflexive dimension of mathematical modeling, it is impossible to work only with theories or techniques that facilitate the solution of mathematical models because they can be memorized and forgotten. This aspect of traditional learning prevents students to have access to creativity, conceptual elaboration, and the development the logical, reflexive, and critical thinking.

However, the critical-reflexive dimension of mathematical modeling is based on the students' autonomy, which aims to facilitate the expansion of world view, the development of autonomous thinking, and to contribute to the full exercise of citizenship. According to this perspective, this dimension of mathematical modeling facilitates the development of competencies, skills, and abilities that necessary for students to play a transformative role in society (Rosa & Orey, 2007).

Final Considerations

The fundamental characteristic of teaching towards critical-reflexive efficiency is the emphasis on the students' critical analysis of problems faced by a member of the contemporary society through the use of mathematical modeling. Another important feature of this kind of teaching is the students' personal reflection about the social elements that underpin the globalized world. Thus, the critical perspective of students in relation to the social conditions that affect their own experiences can help them to identify common problems and collectively develop strategies to solve them (D'Ambrosio, 1990).

This is paradigm that incorporates a type of transformatory learning that aims to create conditions that help students to challenge the worldviews and values that are dominant in our society. Through their experiences, they are able to critically reflect on these experiences in order to develop rational discourse by creating meanings necessary for structural transformation of society (Freire, 2000). This presents a rational transformation because it involves critical analysis of sociocultural phenomena through the elaboration of mathematical models.

Mathematical modeling is therefore a teaching methodology that focuses on the development of critical-reflexive efficiency as engages students in a contextualized teaching-learning process by allowing them to get involved in the construction of the social significance of the world (Rosa & Orey, 2007).

The critical-reflexive dimension of mathematical modeling is based on the comprehension and understanding of reality in which students live by reflection, analysis and critical action on this reality. When we borrow systems from reality, students begin to study them symbolic, systematic, analytical and critically. In this regard, starting from problem situations, students can make hypotheses, test them, correct them, make transfers, generalize, analyze, complete and make decisions about the object under study.

Thus, critical mathematical modeling seeks to explain different ways of work with reality. So, critically reflecting about reality becomes a transformational action that seeks to reduce its complexity by allowing students to explain it, understand it, manage it and find solutions to the problems that arise therein.

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